

LOCAL HEAT TRANSFER ON A PLATE WITH AN  
INITIAL UNHEATED PART WITH LAMINAR FLOW  
IN THE BOUNDARY LAYER

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UDC 536.244

Measurements are reported on local heat transfer for a plate in a current of air with a constant wall temperature in the heat-transfer zone; a water-cooled sectional plate was employed, namely, a calorimeter of length 0.3 m and thickness 0.008 m. The leading edge of the plate took the form of a circle of radius  $R = 0.072$  m, which provided  $2.3 \pm 0.2\%$  turbulence in the incident flow, and also a stable laminar mode of flow in the boundary layer up to a Reynolds number of  $140 \cdot 10^3$ . The air temperature was  $50^\circ\text{C}$ , while the surface temperature of the plate in the heat-transfer zone was  $10^\circ\text{C}$ , with the speed of the air flow in the range 2-35 m/sec. The heat-transfer measurements were made with transducers for two cases: with the plate heated throughout and with the plate having an initial unheated part, whose length was 0.0235, 0.0385, or 0.1635 m.

The results for complete heating agreed within  $\pm 12\%$  with Polhausen's formula. The initial unheated part reduced the heat-transfer rate, and the effect of this was closely described by the correction factor derived by Eckert from the analytical solution.

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HEAT TRANSFER IN PLASTIC-FILM CLOCHES

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The thermal conditions in unheated plastic cloches are determined by the energy accumulated in the soil when night conditions are unfavorable; proper use requires evaluation of the various types of heat loss from such structures. The heat loss from a cloche consists of convective and radiation fluxes from the surface of the soil, those from the surface of the cloche itself, and radiation from surrounding areas.

A previous study has been made [1] of the thermal conditions in such cloches on the basis of convection, radiation, and freezing of the surface of the soil during night frosts; the solutions indicate that the spectral characteristics of the structure influence the balance between the convective and radiative fluxes, with the latter predominant and in any practical case being not less than 70% of the total heat loss.

Figure 1 shows the air temperature  $t_a$  under the film as a function of the spectral characteristics of the cloche, which themselves determine the radiation emission. The relationship is nonlinear and indicates that the transmission coefficient  $D_2$  for long-wave radiation does not by itself fully characterize the thermal protection provided by a cloche.

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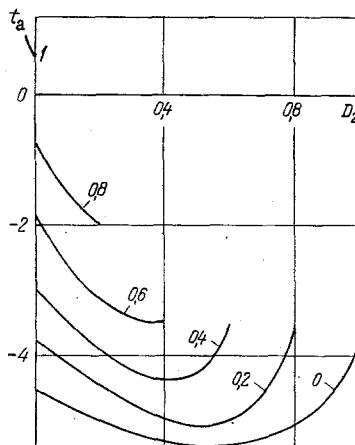


Fig. 1. Air temperature in a cloche as a function of the spectral characteristics of the covering (outside air temperature,  $-6^{\circ}\text{C}$ , thermal conductivity and thermal diffusivity of soil under film,  $0.8 \text{ W/m}\cdot\text{deg}$  and  $3 \cdot 10^{-4} \text{ m}^2/\text{sec}$ ). The numbers on the curves are the values of  $r_2$ . The values for  $t_a$  are in  $^{\circ}\text{C}$ .

The results show that one of the major features for classifying such devices, which are used, for example, to protect plants from frost, is the reflection coefficient  $r_2$  for long-wave radiation.

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Dep. 1921-76, January 26, 1976.  
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#### THERMAL CONDUCTIVITY OF FINE-GRAINED GRAPHITIZED COKE AT $350\text{-}2500^{\circ}\text{K}$

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UDC 536.2.082

Artificial graphite is commonly made from finely divided coke from petroleum and other sources, which is first heat-treated (graphitized) above  $1500^{\circ}\text{K}$ . Reliable values can be obtained for the heat-transfer parameters for a bed of such material after compression on the basis of the experiments reported here. The thermal conductivity was determined for the range  $350\text{-}1300^{\circ}\text{K}$  by the tube method, while the range  $1300\text{-}2500^{\circ}\text{K}$  was covered by using a cylinder with an internal heat source.

Measurements were made of the thermal conductivity of petroleum coke and pyrolysis coke of particle size up to 15 mm in air at pressures of 7-10 Pa and in helium at  $15 \cdot 10^4$  Pa after compression of the powder at pressures up to 16,000 Pa, the results covering the range  $350\text{-}2500^{\circ}\text{K}$ . The temperature dependence of the thermal conductivity was much the same for all compression pressures, which confirmed that the theoretical relationships due to Dul'nev, Kaganer, and others are applicable, and also extends the range of utility of these formulas up to  $1300^{\circ}\text{K}$  inclusive. The measurements also show that the three components of the effective thermal conductivity in a granular material can be measured.

1. The conductivity due to actual contact between grains, which may be measured under high vacuum up to  $500^{\circ}\text{K}$ .
2. The conduction due to radiation from particle to particle via the cavities, which may be determined

under high vacuum above 800°K as the difference between the effective thermal conductivity and the contact component.

3. The conductivity due to the gas between the particles, which may be determined as the difference between the total effective thermal conductivity and the sum of the components due to radiation and contact.

This model for the transport mechanism allows one to examine each of the components separately, especially via detailed measurements of the components in fairly simple experiments. For the same reason, differential equations for the effective thermal conductivity in terms of these components are more realistic and convenient.

The following formula applies for the steady-state effective thermal conductivity along the x coordinate:

$$\lambda_x \frac{\partial T}{\partial x} = \lambda_{cx} \frac{\partial T}{\partial x} + \lambda_{\phi x} \frac{\partial T}{\partial x} + \lambda_{px} \frac{\partial T}{\partial x}$$

and, correspondingly, for the nonstationary thermal conductivity:

$$C \frac{\partial T}{\partial \tau} = \frac{\partial}{\partial x} \left( \lambda_{cx} \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial x} \left( \lambda_{\phi x} \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial x} \left( \lambda_{px} \frac{\partial T}{\partial x} \right).$$

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## THERMOPHYSICAL CHARACTERISTICS OF POLYETHYLENE CONTAINING GLASS FIBER

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A composite containing crosslinked polyethylene filled with glass fiber has been made, which has improved heat resistance in combination with good physicomechanical parameters.

The initial material was low-density polyethylene grade 10702-020, which was filled with 5, 10, 20, or 40 wt. % glass fiber of length 5-10 mm, grade VSO-10V. The crosslinking agent was dicumyl peroxide, MRTU (Interrepublic Technical Specification) 09-6273-69, which was used in the following weight ratios to the polyethylene: 3, 5, and 7 wt. %.

The degree of crosslinking in the polymer has a marked effect on the strength and thermophysical characteristics.

Heating at a constant rate was used to determine the thermophysical characteristics.

The errors of measurement did not exceed the theoretical errors for the method (5% for  $\lambda$  and 8% for  $a$ ).

The results were used to draw up the following curves:

$$\lambda = f(P), a = f(P), c = f(P).$$

The thermal conductivity and thermal diffusivity increase with the degree of filling;  $\lambda$  and  $a$  fall over the range 5-10 wt. % filling, the falls only exceeding the error of experiment slightly, however. The thermophysical parameters of composites containing up to 20 wt. % filler differed little from those of the initial polyethylene. At higher filling contents, the thermal diffusivity and thermal conductivity increased.

The filler reduced the specific heat, the value  $1.6 \cdot 10^3$  J/kg·deg being attained at 40% filling, which is less by almost a factor of 1.5 than that for pure polyethylene, the reason being that the specific heat of glass fiber is less than that of polyethylene.

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DOUBLE-CONTACT METHOD OF DETERMINING  
MASS-TRANSFER COEFFICIENTS

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UDC 66.021.3

It is not practicable to determine by experiment the mass content of a transported material in various solids, granular materials, or bonding agents, which complicates the determination of mass-transport coefficients. In that case, the coefficients can be determined as follows. A specimen with an initial specific mass content  $u_{10}$  is put in contact with a standard specimen and a specimen of the same material but having a different specific mass content  $u_{20}$ . The experiment is performed under isothermal conditions for a time  $\tau$  such that the specific mass contents of all three bodies far from the contacts remain unchanged.

Measurements are made of the amounts of material transferred from body 1 to body 2 and to the standard, namely,  $M_s$  and  $M_s^s$ , respectively.

The amount of material transferred through unit area in time  $\tau$  for bodies 1 and 2 can be put as

$$M_s = (u_{10} - u_{20}) \gamma \sqrt{\frac{a_m \tau}{\pi}}, \quad (1)$$

and for body 1 and the standard

$$M_s^s = 2 (u_{10} - u_b) \gamma \sqrt{\frac{a_m \tau}{\pi}}. \quad (2)$$

From (1) and (2) we have

$$\frac{M_s^s}{M_s} = \frac{2 (u_{10} - u_b)}{u_{10} - u_{20}}. \quad (3)$$

The effective transport coefficient  $a_m$  for the material is found from (1), while (3) gives  $u_b$ .

The mass-transport potential  $\theta_b$  is deduced from the mass content of the standard body at the boundary on the basis of

$$u_b = c_m \theta_b,$$

which enables one to calculate the mean isothermal specific capacity  $c_m$  for the material.

This method has been used to determine transport coefficients for plasticizers in polymers with large amounts of filler. The standard body was a stack of polyethylene films, which provided a simple system as well as reliable determination of  $\theta_b$ .

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GENERALIZED CONSTRUCTION OF ISOTHERM-DISPLACEMENT  
LAWS FOR BOUNDARY BODIES

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The temperature distribution in a body of regular geometrical shape can be represented as a series in even powers of the coordinate:

$$T(x, \tau) = \sum_{n=0}^{\infty} A_{2n}(\tau) x^{2n},$$

and then the rate of displacement  $v_\theta$  for the isotherms is

$$v_\theta = \left[ \frac{\partial (l_0 - x)}{\partial \tau} \right]_\theta = \frac{\sum_{n=0}^{\infty} A'_{2n}(\tau) x^{2n}}{2 \sum_{n=1}^{\infty} n A_{2n}(\tau) x^{2n-1}}. \quad (1)$$

Expressions for  $A_{2n}(\tau)$  have been used to show that the series in the numerator and denominator in (1) converge in accordance with d'Alembert's criterion for constant values of the thermophysical parameters and monotonic time variation  $A_0(\tau)$  in the temperature at the center.

It has also been shown that for  $A_0(\tau) \sim \tau$  (quasistationary states) that

$$v_\theta = \frac{ma}{x}, \quad (2)$$

while for  $A_0(\tau) \sim \exp(-\alpha\tau)$  (regular thermal state) that  $\bar{v}_\theta = [\partial(1 - \eta)/\partial Fo]_\theta$ , respectively, for plate, cylinder, and sphere:

$$\mu_1 \operatorname{ctg} \mu_1 \eta; \mu_1 \frac{J_0(\mu_1 \eta)}{J_1(\mu_1 \eta)}; \frac{\mu_1^2}{\frac{1}{\eta} - \mu_1 \operatorname{ctg} \mu_1 \eta}.$$

In the nonlinear case, where  $c\rho = c\rho(T)$  and  $\lambda = \lambda(T)$ , it is found that

$$A_0(\tau) = T(x=0, \tau); A_2(\tau) = \frac{1}{2ma} A_0'(\tau);$$

$$A_4(\tau) = \frac{1}{(4m+8)a} \frac{dA_2}{d\tau} + \frac{m}{2m+4} A_2^2 \frac{d \ln c\rho}{d\tau} - \frac{1}{2} A_2^2 \frac{d \ln \lambda}{d\tau}.$$

If it is assumed that  $d \ln c\rho/d\tau$  and  $d \ln \lambda/d\tau$  are bounded, the following formula is derived from (1) and the values for  $A_{2n}(\tau)$  for the rate of displacement of the isotherms in the central part of the body ( $x \rightarrow 0$ ):

$$v_\theta = \frac{ma(T)}{x}, \quad (3)$$

where  $a(T)$  is the thermal diffusivity of the material at the temperature applicable to the center at time  $\tau$  for which the speed  $v_\theta$  at point  $x$  near the center of symmetry is to be calculated.

Equation (3) can be used to determine the temperature dependence of the thermal diffusivity  $a = a(T)$  from a single experiment without restriction on the rate of change of the temperature distribution in the body, and thus without relation to the form of the boundary conditions.

#### NOTATION

$T$ , temperature,  $x$  and  $\eta = x/l_0$ ,  $l_0$  the dimensional and dimensionless coordinates of a point in the body, and also the characteristic dimension of the body (half-thickness for a plate, radius of cylinder or sphere),  $c\rho$ ,  $\lambda$ ,  $a = \lambda/c\rho$ , the bulk specific heat, thermal conductivity, and thermal diffusivity,  $\tau$ ,  $Fo = a\tau/l_0^2$ , time and Fourier number;  $\mu_1$ , the first root of the characteristic equation for the thermal conduction.

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## A NONSTATIONARY PROBLEM IN CONVECTIVE DIFFUSION

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UDC 621.38

Distribution of the concentration has been determined for the reducing component in a working electrolyte in the cathode region of a chemotron transducer in the form of a truncated cone. The region is axially symmetrical, so the concentration distribution is two-dimensional if the region is treated as a truncated wedge. In practice, the truncated cones have radii at the ends that differ little, so the wedge may be approximated as a circular sector bounded by the arcs of two concentric circles  $r = a_1$  and  $r = a_2$  ( $a_1 < a_2$ ) and two sections of straight lines constituting an angle  $2\varphi_1$  ( $\varphi_1 > 0$ ).

It is assumed that the readout electrode is the cathode, which coincides with the side surface of the truncated cone, while the electrolyte flows through the cathode channel in a laminar fashion, and the device works in the limiting diffusion-current mode. It is also assumed that the concentration of the active component at the inlet at the cathode channel is  $C_1 = \text{const}$ , whereas it is zero at the outlet, i.e.,  $C(a_2, \varphi, t) = 0$  if  $t > 0$  and  $-\varphi_1 \leq \varphi \leq \varphi_1$ .

In that case, the derivation of the concentration in this region amounts to solving the equation for convective diffusion:

$$\frac{\partial C}{\partial t} + \frac{a_1 v_0}{r} \cdot \frac{\partial C}{\partial r} = D \nabla^2 C \quad (1)$$

subject to the following boundary and initial conditions:

$$\begin{aligned} C(r, \varphi_1, t) = C(r, -\varphi_1, t) = 0, \quad a_1 < r < a_2, \quad t > 0, \\ C(a_1, \varphi, t) = C_1 = \text{const}, \quad -\varphi_1 < \varphi < \varphi_1, \quad t > 0, \\ C(a_2, \varphi, t) = 0, \quad -\varphi_1 < \varphi < \varphi_1, \quad t > 0, \\ C(r, \varphi, 0) = C_0(r, \varphi), \quad a_1 < r < a_2, \quad |\varphi| < \varphi_1. \end{aligned}$$

A Laplace transform is applied to (1) to give

$$D \left[ \frac{\partial^2 \bar{C}}{\partial r^2} + \frac{1-2\nu}{r} \cdot \frac{\partial \bar{C}}{\partial r} + \frac{1}{r^2} \cdot \frac{\partial^2 \bar{C}}{\partial \varphi^2} \right] - p \bar{C}(r, \varphi, p) = -C_0(r, \varphi), \quad (2)$$

where

$$2\nu = \frac{a_1 v_0}{D}$$

The substitution  $\bar{C} = r^\nu U(r, \varphi, p)$  transforms (2) to

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial U}{\partial r} + \frac{1}{r^2} \cdot \frac{\partial^2 U}{\partial \varphi^2} - \left( \frac{\nu^2}{r^2} + \frac{p}{D} \right) U = -\frac{C_0(r, \varphi)}{Dr^\nu}. \quad (3)$$

It is shown that a special integral transform applied to (3) with the boundary conditions appropriately transformed can give the exact solution

$$U(r, \varphi, p) = \frac{2}{\pi^2} \int_0^\infty F(\varphi, p, s) \frac{R_n \left( \sqrt{\frac{p}{D}} r, p, s \right) s \operatorname{sh} \pi s}{\left\| J_{is} \left( \sqrt{\frac{p}{D}} a_2 \right) \right\|^2} ds,$$

where  $F(\varphi, p, s)$ ,  $R_n(\sqrt{p/D}r, p, s)$  are known functions and  $J_{is}$  is a Bessel function of complex argument; then  $\bar{C} = r^\nu U(r, \varphi, p)$ , and thus the Riemann-Mellin formula gives

$$C(r, \varphi, t) = \frac{1}{2\pi i} \int_L \bar{C}(r, \varphi, p) e^{pt} dp.$$

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## NONSTATIONARY MOTION OF A FLUID IN A CYLINDRICAL SHELL WITH AN ADDITIONAL NOISE SOURCE

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UDC 532.51

Differential equations have been solved for the nonstationary motion of a fluid in an axially symmetrical shell of rotation neglecting the frictional loss; these characterize the process with sufficient precision only for a shell having no additional noise sources at any point along the length, such sources being leaks, blind branches, partly blocked pressure controls, and so on.

These additional sources of interference substantially distort the pattern of the nonstationary processes; interference then occurs between oscillations of various frequencies and amplitudes, which can cause automatic-control or monitoring devices to trigger, and can also cause overshoot and so on.

Nonstationary motion of a fluid in a cylindrical shell with an additional noise source is considered for the case of instantaneous occurrence of a perturbation (for instance, connection or disconnection of a pump, or opening or closure of a pressurizing system).

It is shown that the system of equations for the nonstationary motion in the presence of this additional noise and simple transformations produce an equation with a deviating argument of neutral type.

The case is that of a noise source producing a constant delay  $\tau$ .

It is assumed that the speed of propagation  $c$  for the perturbations is constant. Mikusinsky transformations are used for zero initial conditions and for boundary conditions specified as piecewise-continuous time functions to derive solutions for the equation without delay and with delay. An explicit operator representation is used to examine the behavior of the solutions. It is found that the liquid in any given section is involved in several different types of motion, the number of these increasing with time. Oscillations arising in the initial section propagate with the velocity  $c$  throughout the length. Distorted oscillations begin to propagate for  $\tau < \infty$  after time intervals  $t = j\tau$  ( $j = 1, 2, \dots$ ), i.e., noise effects arise. These oscillations also propagate with speed  $c$ , and at an arbitrary point  $x$  show distortions of a definite form. The propagating waves are reflected from the ends, and the distortion remains the same for any point in the range for all reflections.

A comparatively simple formula describes the nonstationary motion in a cylindrical shell with such an additional noise source (leaks, blind sections, and so on), and this can be used as a working formula.

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## AN ITERATIVE METHOD IN OPTIMAL CONTROL OF METAL HEATING

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UDC 669.046.518.61

Optimal control in a metal heater involves a system with distributed parameters, no matter what the performance criterion, and the system is described by differential equations in partial derivatives in conjunction with complex boundary and initial conditions.

Numerical methods have to be used in such cases, but the lack of suitable computational techniques has previously prevented one from utilizing numerical methods fully, in particular, iterative methods. Fast digital computers incorporated into automatic-control systems for metal heating can provide adequate accuracy in defining the optimal control input by iterative methods, one advantage of these being that the operations are always of the same type, and therefore are readily programmed.

Quasilinearization is used as an iterative method in optimal control for metal heating in a system with distributed parameters; the method is as follows. The differential equation for the controlled process is used with the boundary conditions which are supplemented by equations for the influence functions (conjugate system). Nominal values are then selected for the phase variables and influence functions to satisfy as many of the boundary conditions as possible, and the optimality conditions are used to determine the nominal control action. The equations for the controlled process and the conjugate system are linearized with respect to this nominal input, after which a sequence of inhomogeneous two-point boundary-value problems is solved. The solution is improved until the equations for the heating and those in the conjugate system are satisfied with the required accuracy.

This method is applicable to define the most rapid mode of heating, and expressions are derived that can be utilized in the iteration algorithm to define the optimal solution.

This method can also be used in minimizing the energy consumption in heating, minimizing scaling, and so on.

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A method is given for determining two-dimensional nonstationary temperature distributions in a multi-layer system.

The system consists of  $n$  homogeneous plane-parallel layers of arbitrary thickness that differ in thermal diffusivity. Each layer has its own local coordinate system  $x_k O y_k$ , with the  $x_k$  axis taken as the symmetry axis of the layer and the  $y_k$  axis lying along a common straight line perpendicular to the  $x_k$  axes. The layers are numbered from the top downward.

The temperature distribution in any layer is defined by the function  $T_k(x, y, t)$ , which is a solution to the equation for nonstationary thermal conduction. Laplace and Fourier integral transforms with respect to  $t$  and  $x$ , respectively, are applied in sequence to reduce this equation to an ordinary second-order differential equation with constant coefficients for the desired function. The general solution is dependent on two arbitrary coefficients  $A_k$  and  $B_k$ , while the solution for the system as a whole is dependent on  $2n$  coefficients, which can be derived from the boundary conditions and the conditions for temperature and heat-flux continuity at the common boundaries. The continuity conditions provide recurrence formulas for the coefficients  $\alpha_k$  and  $\beta_k$ , which are equivalent to the unknown coefficients  $A_k$  and  $B_k$ . These recurrence formulas define all the functions  $\alpha_k$  and  $\beta_k$  and, consequently, all the  $\bar{T}_k(\alpha, y, p)$  if one can define a single pair of the  $\alpha_k$  and  $\beta_k$ , for instance  $\alpha_1$  and  $\beta_1$ .

The values for  $\alpha_1$  and  $\beta_1$  have been determined for major boundary-value cases for such a system, which thus yield the other coefficients via the recurrence formulas, which involves solving only one linear algebraic equation, no matter what the number of layers. For instance, if the following conditions are specified for the upper and lower boundaries of the system,

$$T_1(x, h_1, t) = f(x, t), \quad \frac{\partial T_n(x, -h_n, t)}{\partial y} = 0, \quad (1)$$

then the expressions for  $\alpha_n$  and  $\beta_n$  are used with (1) in terms of the coefficients  $\alpha_1$  and  $\beta_1$  to give

$$\beta_1 = H_n(\alpha, p, h_1, h_2, \dots, h_n) \alpha_1,$$

where  $h_1, h_2, \dots, h_n$  are the half-thicknesses of the corresponding layers and  $H_n$  is the function characterizing the thermophysical parameters of the system. This also shows that a single equation for  $\alpha_1$  is sufficient to solve the problem for the transform region.

The temperature distribution in the multilayer system can be derived via inversion formulas when  $\alpha_k$  and  $\beta_k$  have been derived.

Computational difficulties are involved in deriving  $H_n$ , but these may be avoided by employing a recurrence formula for the function.

The application to a two-layer system is discussed.

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## DROPLET SIZE IN MONODISPERSE LIQUID SPRAYING

Approximately identical drops are produced when a nonwetting liquid flows at a very low rate from a capillary under gravity; the drop diameter  $d = (12R\sigma/\rho g)^{1/3}$  given by theory is confirmed by experiment, e.g., for drops of water formed at the lower end of a vertical steel capillary of radius  $R = 0.024$  or  $0.033$  cm. About 30 drops were produced in 1 min. The ratio of the mean measured  $d$  to the calculated value was  $0.89-0.92$ , i.e., the agreement was close.



When a nonwetting liquid is dispersed by a rotating perforated drum (at very low liquid flow rates),  $d = (12R\sigma/\rho r\omega)^{1/3}$ , and experiments on this were based on the drum of outside radius  $r = 2.5$  cm and radius for a single radial hole  $R = 0.025$  cm. Water was supplied at a flow rate  $Q = 0.004$  cm<sup>3</sup>/sec with the drum rotating at  $\omega = 157$  sec<sup>-1</sup>. The mean values of  $d$  differed from the calculated values by 9-10%. The coefficients of variation for the deviations of  $d$  from the mean were 0.84-1.5%, i.e., the drops were highly uniform in size.

In the case of a wetting liquid dispersed by a rotating disk  $d = (C/\omega)(\sigma/\rho r)^{1/2}$ , with the constant  $C$  close to 2.7 for mineral oils [1]. The formula has been verified for  $\omega$  in the range from 30 to 10,000 sec<sup>-1</sup>,  $r$  from 2 to 11 cm,  $\rho$  from 0.9 to 13.6 g/cm<sup>3</sup>,  $\sigma$  from 29 to 465 g/cm<sup>2</sup>,  $\eta$  from 0.01 to 26 g/cm·sec, and  $d$  from 0.003 to 0.4 cm. In this range,  $C$  varies from 1.9 to 4.6 [1-3]. The formula is also applicable for a wetting liquid dispersed by a rotating perforated drum (this has been verified for  $\omega$  of 105-314 sec<sup>-1</sup>,  $r$  of 1-5 cm,  $R$  of 0.04-0.2 cm,  $\sigma$  from 29 to 33 g/sec, and  $\eta$  of 0.19-2.4 g/cm·sec). Throughout this range, the values of  $C$  for drum and disk were similar.

These formulas for the droplet sizes for a variety of dispersal processes thus give results in total agreement with experiment, i.e., are suitable for approximate calculations.

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#### SATELLITE DROPLETS IN MONODISPERSE DROPLET PRODUCTION

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UDC 66.769.8

When a liquid is being dispersed as droplets of the same size, smaller polydisperse satellite droplets are sometimes produced.

A liquid emerging from a capillary under gravity at very low flow rates (primary state) produces only identical droplets (no satellites); satellite droplets are produced only at elevated flow rates (near the critical flow rate and in the second mode of spraying).

In the case of a rotating sprayer, the auxiliary droplets are formed even in the first state, i.e., at very low flow rates. Experiments have been performed on liquid spraying at very low flow rates, and even a rate of  $Q = 0.00027$  cm<sup>3</sup>/sec did not yield droplets without satellites, but in that case the main droplets and the satellites had a very narrow range of sizes, with the number of satellites equal to the number of main droplets, the diameter ratio being about 0.3 and the proportion of satellites by weight about 3%.

The spread in the droplet sizes increases with  $Q$  (the width of the deposition ring increases).

Similar results have been obtained in dispersing a liquid with rotating perforated drums.

The probable cause of the differences is that droplets produced by gravity alone grow in virtually immobile air, whereas centrifugal dispersal results in large speeds of the dispersing system with respect to the air, i.e., large air-resistance forces act on the droplets. It is clear that at the start of formation, where the droplet projects only a little beyond the outer edge of the dispersal system, the air resistance should be of secondary importance, but in the second (final) stage the air resistance increases, and this would appear to accelerate the second stage, i.e., droplet detachment, which implies accelerated rupture in the neck between the drop and the liquid at the edge (by comparison with immobile air), with the result that satellite droplets are formed.

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## HYDRODYNAMIC BEHAVIOR OF A TWO-PHASE VORTEX FLOW AT LOW VORTICITIES

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UDC 532.529.5

A mathematical model is presented for the hydrodynamic processes in an absorption apparatus employing a rising two-phase spiral flow.

In such a case, the laminar liquid film is exposed to a turbulent spiral gas flow, and it is assumed that the liquid and gas are incompressible, with the flow in the steady state and axially symmetrical. The film thickness is much less than the diameter of the contact device, so the problem is one involving the differential equations for a boundary layer, which can be considered as lying on a flat surface.

The viscous tangential stress is accompanied by turbulent stresses in the gas flow, and these can be discussed in terms of available measurements. The mathematical model is described by a system of equations for the motion and for the continuity for the turbulent gas flow and for the laminar film, which are supplemented by boundary conditions for the immobile surface (attachment condition), for the interface (equality of the velocities and tangential stresses, and absence of mass transfer through the interface), and also for the flow (constancy of the axial component of the velocity). The film thickness may be determined by employing the equation for the constancy of the liquid flow rate in each section.

The problem has been solved with a Minsk-22 computer.

The tangential, axial, and radial components of the velocity in the film and the boundary gas layer have been determined in relation to the actual speed of the gas, the liquid input, and the angle of the spiral; values have also been derived for the tangential stress at the interface, the film thickness, and other quantities, which are presented in figures and a table.

For instance, for a liquid flow rate  $G = 0.115$  kg/msec, an axial speed  $u_{ax} = 10.75$  m/sec, and a film thickness ranging from  $\delta = 0.75 \cdot 10^{-4}$  m at  $x = 0.075$  m to  $\delta = 1.8 \cdot 10^{-4}$  m at  $x = 0.078$  m will give a tangential stress  $\tau_{\delta}$  under analogous conditions ranging from 5 to 2.5 kgf/m<sup>2</sup>.

These results show that a spiral flow differs from an axial flow in giving high velocity gradients near the interface, which accelerates the mass transfer.

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## NONSTATIONARY THERMOELECTRIC COOLING IN A TWO-STAGE THERMOELEMENT

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UDC 537.324

Experimental and mathematical studies have been made on nonstationary thermoelectric cooling in a two-stage thermoelement in the constant-current mode with separate supplies to the stages.

The thermoelements were made as previously described [1, 2]; the height and area of a branch in the working section WS were 1 cm and 0.01-0.05 cm<sup>2</sup>, while those for the rear stage RS were 4-65 cm and 2 cm<sup>2</sup>. Experiments have been made with the independent and joint operation of the stages. If only the rear stage carries current, the peak cooling in the working stage, which here is a passive load, does not attain the optimal cooling for a single thermoelement in the steady state ( $\Delta T_{opt}$ ); the maximum cooling in that case ( $\Delta T \cong 0.6 \Delta T_{opt}$ ) occurs when the rear stage carries a current  $2I_{opt}$  ( $I_{opt}$  is the optimum current). If only the working stage carries current, the rear stage serves as a heat sink, in which case the peak and steady-state degrees of cooling attained for  $I_{opt}$  are the largest, the values being, respectively,  $0.7 \Delta T_{opt}$  and  $0.6 \Delta T_{opt}$ . If the two stages are operated together, the cooling is dependent not only on the ratio between the currents in the

stages but also on the connection sequence. The largest temperature reduction ( $\sim 1.2\Delta T_{\text{opt}}$ ) occurs for  $I_{\text{RS}} = 2I_{\text{opt}}$  and  $I_{\text{WS}} = I_{\text{opt}}$ . Here the current in the working stage is switched on at an instant such that the minimum temperatures due to the two currents are attained simultaneously.

The Laplace transform has been applied to derive analytical expressions for the cold-junction temperature in the working stage when current flows only in one of the stages, and also when the two stages work together. If the rear stage alone is operating, the temperature at the cold junction in the working stage is dependent on the current. If the current is small ( $I_{\text{RS}} < 2I_{\text{opt}}$ ), the maximum cooling at the cold junction in a two-stage thermoelement,  $\Delta T_{\text{opt}}$ , is  $\Pi^2/2\rho\kappa$ , which corresponds to  $I_{\text{opt}}$  and is attained in the steady state. In the range  $I_{\text{RS}} > 2I_{\text{opt}}$ , the maximum cooling is a peak effect and occurs at small times. The peak cooling  $\Delta T_{\text{m}}$  in that case is dependent on the ratio of the areas of the stages  $\beta = (S_{\text{RS}} - S_{\text{WS}})/(S_{\text{RS}} + S_{\text{WS}})$  and on the parameter  $h/2\sqrt{at_{\text{m}}}$  ( $h$  is the height of the working stage,  $a$  is the thermal diffusivity of the material, and  $t_{\text{m}}$  is the time at which the maximum cooling is attained). The value of  $\Delta T_{\text{m}}$  increases as  $h/2\sqrt{at_{\text{m}}}$  decreases and as  $\beta$  increases, but it does not exceed  $\Pi^2/\pi\rho\kappa$  ( $\Pi$  is the Peltier coefficient,  $\rho$  is specific resistance, and  $\kappa$  is thermal conductivity). If the working stage alone carries current, it operates as a finite body if the height  $h$  is small or if the time is large, and the heat conduction is then dependent on the heat-transfer conditions at the hot end. If the area of the working stage is much less than that of the rear stage, then the Peltier and Joule components of the heat are effectively transferred to the rear stage, and then the working stage behaves as a single thermoelement whose hot-junction temperature is maintained constant. The maximum steady-state cooling in the device is  $\Delta T_{\text{opt}} = \Pi^2/2\rho\kappa$ . If the areas of the stages are comparable, the heat transfer through the rear stage is insufficient, and the Peltier heat at the hot junction in the working stage and the Joule heat in the volume of the device generally together reduce the limiting cooling attained in the working stage. If the two stages operate together, the peak cooling is maximal if the two components themselves are maximal, since the times at which the peak coolings are attained at the cold junction of the working stage due to each stage separately are not the same, in which case the currents must be applied to the stages not simultaneously but at instants such that the peak coolings coincide in time. In that case, the maximum value for the peak cooling  $[1 + (2/\pi)(\pi^2/2\rho\kappa)]$  is about 0.8 of the limiting cooling produced by a two-stage thermal element in the steady state.

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#### SOLUTION OF A NONLINEAR EQUATION FOR THE SWELLING OF A CYLINDRICAL FUEL ROD

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UDC 518:539.3

A numerical method is presented for solving the nonlinear equation for swelling in a cylindrical fuel rod; a model for gas swelling is used, which involves the assumption that the nuclear fuel consists of regularly disposed identical spherical cells in the initial state, each of which is a thick-walled hollow sphere. Any elementary volume of the body contains a reasonably large number of spherical cells.

The swelling in the individual cells is determined by the creep in the thick-walled hollow spheres, which are loaded by internal pressure from the gaseous fission products, which accumulate in the free volume as the fuel is consumed, while the external pressure is determined by the interaction between the individual swelling elements and is equal to the component of the spherical macrostress tensor arising from creep in the fuel core.

The problem is that of numerical solution of a nonlinear equation for  $S(\rho, t)$ :

$$F_{\alpha}(\rho) \left[ \frac{1}{(S(\rho, t) + \varepsilon)^{1/m}} - \frac{1}{(S(\rho, t) + 1)^{1/m}} \right] \left( \frac{\partial S(\rho, t)}{\partial t} \right)^{1/m} = \frac{W(\rho, t) t}{S(\rho, t) + \varepsilon_0 - lt} +$$

$$+ \frac{1}{2} \varphi(\rho, t) C(t) + \varphi(\rho, t) \frac{\partial \psi(\rho, t)}{\partial t} - \frac{2}{3} \varphi(\rho, t) \frac{\partial S(\rho, t)}{\partial t} - \int_{\rho}^1 \varphi(\rho', t) \left[ 2 \frac{\partial \psi(\rho', t)}{\partial t} - \frac{\partial S(\rho', t)}{\partial t} \right] \frac{d\rho'}{\rho'};$$

$$C(t) = \frac{\int_0^1 \varphi(\rho', t) \frac{\partial S(\rho', t)}{\partial t} \rho' d\rho'}{3 \int_0^1 \varphi(\rho', t) \rho' d\rho'}; \quad \frac{\partial \psi(\rho', t)}{\partial t} = \frac{1}{\rho^2} \int_0^{\rho} \rho' \frac{\partial S(\rho', t)}{\partial t} d\rho';$$

$$\varphi(\rho, t) = \frac{F_1(\rho)}{\left[ 3 \left( 2 \frac{\partial \psi(\rho, t)}{\partial t} - \frac{\partial S(\rho, t)}{\partial t} \right)^2 + \left( \frac{3}{2} C(t) - \frac{\partial S(\rho, t)}{\partial t} \right)^2 \right]^{\frac{m-1}{2m}}}.$$

The method consists in selecting a parameter  $\varepsilon^{(0)}$  such that the norm of the finite-difference operator becomes much less than 1, which provides rapid convergence in the successive approximations. Then  $\varepsilon^{(n)} = C^{(n-1)} + \delta\varepsilon$  is adjusted to give the required values of  $\varepsilon$  by successive approximation in the first time step, the finite-difference equation for  $\varepsilon^{(n)}$  being solved by means of the values of the corresponding functions obtained for  $\varepsilon^{(n-1)}$ . The solution to the nonlinear swelling equation for the subsequent steps is then found by using the solution for the previous step as the zeroth approximation in the next step.

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## TEMPERATURE DISTRIBUTION AND THERMAL STRESS IN THE WALL OF AN INSULATED PIPELINE

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An approximate analytical solution is presented for the internal conjugate problem for nonstationary heat transfer in an insulated pipeline subject to periodic variations in the temperature of the fluid at the inlet. Thermal stresses with a cyclic time course arise in the wall, which can cause the material to fail from thermal fatigue.

The temperature distribution has been derived by operational methods by approximate expansion of the exponential in the image region:

$$\exp z \approx \frac{1 + z/2}{1 - z/2}.$$

The thermal stresses in the wall are determined from standard expressions from the quasistatic theory of thermoelasticity on the assumption of a planar state of stress in the pipeline wall.

A detailed analysis of the temperature distributions and thermal stresses in the stationary-periodic stage is presented.

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## INTEGRAL REPRESENTATION OF SOLUTIONS TO NONCLASSICAL THERMAL-CONDUCTION PROBLEMS

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UDC 536.24.02

Considerable computational difficulties are involved with two- and three-dimensional problems in thermal conduction for multicoupled regions of complex shape; these difficulties increase if there are nonuniformities

(for instance, stratified bodies). Such problems arise in many aspects of science and technology, so efficient algorithms are of some considerable interest.

Two-dimensional problems in steady-state thermal conduction are considered for a multilayer strip weakened by holes of arbitrary shape, and also a one-layer strip with a finite number of holes; an algorithm is described that uses integral representation of the target quantities, which is applicable also to a three-dimensional layer with cavities.

In the first stage, the Green's matrix is constructed for the boundary-value problem for a multilayer strip. The boundary conditions at the contact lines presuppose ideal thermal contact, while the boundary conditions at the outer surfaces may take any form. The Green's matrix is constructed by trigonometric expansion for one direction, with subsequent variation of the arbitrary constants, which results in systems of ordinary differential equations. The solution is sought as a finite sum of contour integrals over the boundaries of the holes. The kernel of this integral representation is the Green's matrix, which means that the initial differential equations can be satisfied exactly. The weights of the integrals are determined by satisfying the boundary conditions at the edges of the holes. This results in systems of integral equations of Fredholm type, which are readily solved, for instance, by quadrature-formula methods.

Numerical examples are given of realizations of this algorithm yielding detailed temperature distributions, which are presented as isotherms in figures. The computations were performed with an M-222 computer, the time needed to determine the steady-state temperature distribution in the strip varying from 5 to 10 min.

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